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COULOMB EXCITATION IN DEFORMED NUCLEI

BY

HANS LÜTKEN AND AAGE WINTHER



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Kommissionær: Ejnar Munksgaard

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Errata:

In Table 2 (p. 20) replace the three table headings by

$$\chi^{(1)}(\vartheta, \xi)/\chi^{(1)}$$

$$\chi^{(2)}(\vartheta, \xi)/\chi^{(2)}$$

$$\chi^{(3)}(\vartheta, \xi)/\chi^{(3)}$$

as indicated in the figure caption.

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Synopsis

In multiple Coulomb excitations of deformed nuclei one may observe also a weaker excitation of rotational bands which are associated with states of different intrinsic structure. In the present work, the excitation amplitudes of such states have been computed in the approximation where one neglects the energy differences between the states of a rotational band. For the case of dipole, quadrupole and octupole excitations the results are given in the form of tables. They show that the relative population of the states within a band depends strongly on the spin and K quantum numbers as well as on admixtures in the wave function of components from the ground-state band. A detailed investigation of the dependence of the cross sections on scattering angle is presented. One finds here appreciable deviations from the so-called $\chi(\vartheta)$ -approximation, especially concerning the excitation of the unnatural parity states.

I. Introduction

In recent years it has been possible by means of accelerated heavy ions to perform multiple Coulomb excitation in deformed nuclei to high lying rotational states^{1, 2)}. In these nuclei one may also excite states which differ from the ground state by having a different intrinsic structure³⁾. The excitation probabilities of these states are in general so small that one is justified in describing the mechanism of their excitation by a perturbation treatment⁴⁾. Such a treatment is complicated by the fact that many states in the ground-state band as well as in the rotational band associated with the final state are involved in the excitation process. Even though we consider the transition between the two intrinsic structures only to first order, we must take into account that this transition may take place between a number of different rotational members of the two bands; the virtual excitation preceding as well as the virtual excitation or de-excitation succeeding the transition between the bands must be treated as multiple Coulomb excitations.

It was shown earlier⁴⁾ how one may compute the excitation amplitudes of such states in the impact approximation, neglecting the energy differences between the nuclear states. Computations were made for backward scattering, and the results were generalized to other deflection angles by means of the so-called $\chi(\vartheta)$ -approximation.

In the present work we generalize these calculations in three respects. Firstly, we include the possibility of Coulomb excitations of multipolarities other than quadrupole. Secondly, we discuss the effect of impurities in the rotational bands. Finally, we include in the calculations the effects of a finite energy difference between the intrinsic nuclear states. While in the sudden approximation the two first generalizations are quite trivial, the last generalization introduces a number of complications in the theory. It is thus necessary to consider the deviations from the $\chi(\vartheta)$ approximation as revealed e.g. in the non-negligible excitation of "unnatural" parity states. More serious complications are associated with the computation of the excitation of impure rotational bands, and it has here proved quite convenient to formulate the deviations from the pure rotational model in terms of a modification of the nuclear multipole moments⁷⁾.

In Chapter II we give a formulation of the theory of Coulomb excitation in deformed nuclei, together with a summary of the theory of small deviations from the pure rotational model.

The application of the theory to the excitation of pure rotational states is discussed in Chapter III. One finds here that the total excitation probability of a given rotational band is identical to the result of the ordinary first order perturbation theory. The relative excitation probabilities are determined by simple functions of the intrinsic quadrupole moment, which functions are tabulated in Appendix 2. These functions also depend strongly on the spin and K -quantum numbers of the states, and a measurement of the relative probabilities thus offers a convenient method of determining these quantities.

Finally, in Chapter IV, we consider the effect of weak band mixing. We evaluate explicitly the correction to be applied to the results of Chapter III for the important case of quadrupole excitations of bands which show admixture with the ground-state band.

II. General Theory

In the semi-classical approximation the Coulomb excitation amplitude of a state $|f\rangle$ from the state $|i\rangle$ is given by the expression⁴⁾

$$\alpha_f = \langle f | T e^{-\frac{i}{\hbar} \int_{-\infty}^{\infty} \tilde{\mathfrak{H}}(t) dt} | i \rangle. \quad (1)$$

In this formula T stands for the time ordering operator. The operator $\tilde{\mathfrak{H}}(t)$ is given by

$$\tilde{\mathfrak{H}}(t) = e^{\frac{i}{\hbar} H_0 t} \mathfrak{H}(t) e^{-\frac{i}{\hbar} H_0 t} \quad (2)$$

in terms of the time dependent Coulomb interaction $\mathfrak{H}(t)$ between the nucleus and the projectile and the Hamiltonian H_0 of the free nucleus.

In the present investigation we study the excitation of nuclei with a non-spherical equilibrium shape. For such nuclei the Hamiltonian H_0 is conveniently written in the form⁵⁾

$$H_0 = H_{\text{int}} + H_{\text{rot}} + H_c. \quad (3)$$

The first term is the intrinsic Hamiltonian which describes the motion of the nucleons in the intrinsic coordinate system defined by the shape of the nucleus. In the intrinsic motion are included vibrations of the nucleus around the equilibrium shape. The second term represents the energy associated with the rotation of the intrinsic system and is an operator that only depends on the Eulerian angles which specify the orientation of the intrinsic coordinate system. The last term represents the coupling between the intrinsic motion and the rotation.

The nuclear states $|i\rangle$ and $|f\rangle$ are eigenstates of H_0 . In the pure rotational model one neglects the term H_c and the eigenstates may then be written as the product of eigenstates of H_{int} and of H_{rot} . For an axially symmetric nuclear shape this product wave function is given by

$$|IKM\rangle|nK\rangle = \sqrt{\frac{2I+1}{4\pi}} D_{MK}^I(\alpha, \beta, 0) |nK\rangle. \quad (4)$$

These wave functions represent the rigid rotation with total angular momentum I and magnetic quantum number M of a system described by the intrinsic wave function $|nK\rangle$. The quantum number K denotes the component of the intrinsic angular momentum along the symmetry axis, while α and β are the Eulerian angles specifying the orientation of this axis.

The actual wave functions for the states in pure rotational bands differ from (4) only through a symmetrization with respect to the sign of K . The correct rotational wave functions are found to be⁵⁾

$$|IKMn\rangle = \left\{ \begin{array}{ll} |I0M\rangle|n0\rangle & \text{for } K=0 \\ \frac{1}{\sqrt{2}} \{ |IKM\rangle|nK\rangle + (-1)^{I+K} |I-KM\rangle|n-K\rangle \} & \text{for } K>0 \end{array} \right\} \quad (5)$$

where the intrinsic wave function $|nK\rangle$ determines the parity of the states. For $K=0$ only states of even I or only states of odd I occur.

It is well known from nuclear spectroscopy that even in strongly deformed nuclei there are deviations from the pure rotational model. These deviations must be ascribed to the presence of the coupling term H_c . While the explicit form of the Hamiltonian H_c depends on the details of the nuclear model, one may argue that the most important parts of H_c can be obtained from a series expansion in powers of the rotational angular momentum⁷⁾. Utilizing the invariance and symmetry principles appropriate for an axial symmetric nuclear shape one obtains to second order

$$H_c = h_1 I_- + h_{-1} I_+ + h_2 I_-^2 + h_{-2} I_+^2 + h_0 I^2. \quad (6)$$

In (6) h_i denotes an operator in the intrinsic degrees of freedom which changes the K -quantum number by i units. The operator I denotes the total angular momentum while I_+ and I_- refer to the components of this quantity in the intrinsic coordinate system.

The effect of the interaction (6) may be taken into account by a perturbation treatment. One finds then that the perturbed states may be written in the form

$$|nIKM\rangle_m = e^{iS} |nIKM\rangle. \quad (7)$$

The intrinsic matrix elements of the hermitian operator S are here given by

$$\left. \begin{array}{l} \langle n' \pm (K+1) | S | n \pm K \rangle = \pm i \varepsilon_1 I_{\pm} \\ \langle n' \pm (K+2) | S | n \pm K \rangle = i \varepsilon_2 I_{\pm}^2 \\ \langle n' \pm K | S | n \pm K \rangle = i \varepsilon_0 I^2 \end{array} \right\} \quad (8)$$

where ε_i is the matrix element of h_i divided by the energy difference between the intrinsic states $|n'\rangle$ and $|n\rangle$ ⁷⁾.

In the computation of Coulomb excitation amplitudes we neglect the energy differences associated with H_{rot} and H_c . The operator $\tilde{\mathfrak{H}}(t)$ is thus given by

$$\tilde{\mathfrak{H}}(t) = e^{\frac{i}{\hbar} H_{\text{int}} t} \mathfrak{H}(t) e^{-\frac{i}{\hbar} H_{\text{int}} t}. \quad (9)$$

In the present investigation we deal with a situation in which the interaction $\mathfrak{H}(t)$ may be written as a sum of two terms: A large term $\mathfrak{H}_0(t)$ which represents the interaction with the intrinsic quadrupole moment and a small term $\mathfrak{H}_1(t)$ which represents a residual interaction which may cause transitions to other intrinsic states. Under these circumstances one may perform a series expansion of the exponential function of (1) in powers of the quantity $\mathfrak{H}_1(t)$. To first order in $\mathfrak{H}_1(t)$ one finds

$$\left. \begin{aligned} a_f &= \langle f | e^{-\frac{i}{\hbar} \int_{-\infty}^{\infty} \mathfrak{H}_0(t) dt} | i \rangle \\ &\quad - \frac{i}{\hbar} \langle f | \int_{-\infty}^{\infty} dt e^{-\frac{i}{\hbar} \int_t^{\infty} \mathfrak{H}_0(t') dt'} \tilde{\mathfrak{H}}_1(t) e^{-\frac{i}{\hbar} \int_{-\infty}^t \mathfrak{H}_0(t') dt'} | i \rangle. \end{aligned} \right\} \quad (10)$$

We have here left out the time ordering in the exponential functions since $\mathfrak{H}_0(t)$ only gives rise to transitions within rotational bands and therefore, according to the assumption (9), $\tilde{\mathfrak{H}}_0(t)$ is equal to $\mathfrak{H}_0(t)$.

The Coulomb interaction, $\mathfrak{H}_0(t)$, with the quadrupole moment of the deformed nucleus may be written in the form

$$\mathfrak{H}_0(t) = Z_1 e \sqrt{\frac{\pi}{5}} Q_0 \sum_{\mu} r_p^{-3} Y_{2\mu}(\vartheta_p, \varphi_p) D_{\mu 0}^{2*}(\alpha, \beta, 0), \quad (11)$$

where $Z_1 e$ is the charge and r_p , ϑ_p and φ_p are the coordinates of the projectile, while Q_0 is the intrinsic quadrupole moment of the ground state.

The residual interaction $\mathfrak{H}_1(t)$ is then given by

$$\mathfrak{H}_1(t) = \mathfrak{H}(t) - \mathfrak{H}_0(t) \quad (12)$$

with

$$\mathfrak{H}(t) = 4\pi Z_1 e \sum_{\lambda\mu} (2\lambda+1)^{-1} r_p^{-\lambda-1} Y_{\lambda\mu}(\vartheta_p, \varphi_p) \mathfrak{M}^*(E\lambda, \mu), \quad (13)$$

where $\mathfrak{M}(E\lambda, \mu)$ is the nuclear electric multipole moment of order λ . It is noted that the operator $\mathfrak{H}_1(t)$, besides causing transitions between different intrinsic states, also may describe effects from a small variation of the intrinsic quadrupole moment through the bands or from one band to another.

III. The Excitation of Pure Bands

In this chapter we consider the Coulomb excitation in the pure rotational model. The nuclear wave functions are here given by (5). We first compute the excitation amplitude with the product wave functions (4) and only in the end of the chapter consider the effects of the symmetrization.

For the computation of the excitation amplitudes (10) it is convenient to express the multipole moments in (13) in the form

$$\mathfrak{M}(E\lambda, \mu) = \sum_{\nu} D_{\mu\nu}^{\lambda}(\alpha, \beta, 0) \mathfrak{M}_{\text{int}}(E\lambda, \nu) \quad (14)$$

in terms of the corresponding multipole moments $\mathfrak{M}_{\text{int}}(E\lambda, \nu)$ in the intrinsic coordinate system. In the pure rotational model these moments are independent of the Eulerian angles.

The excitation amplitude of a state in the ground-state band is then determined solely by the first term of (10). Similarly, the second term in (10) determines the excitation of states where the intrinsic wave functions differ from that of the ground state. Since the operators $\mathfrak{S}_0(t)$ and $\mathfrak{S}_1(t)$ commute according to (11)–(14), the excitation amplitude from the state $|I_i K_0 M_i\rangle |n_0 K_0\rangle$ to the state $|I_f K_1 M_f\rangle |n_1 K_1\rangle$ may be written

$$b_{I_f M_f K_1 K_0} = -i \sum_{\lambda} \langle I_f K_1 M_f | k_{\lambda} e^{-ik_0} | I_i K_0 M_i \rangle \quad (15)$$

with

$$\left. \begin{aligned} k_0 &= \frac{1}{\hbar} \int_{-\infty}^{\infty} \mathfrak{S}_0(t) dt \\ &= \frac{Z_1 e}{\hbar} \sqrt{\frac{\pi}{5}} Q_0 \sum_{\mu} S_{2\mu}(\vartheta, 0) D_{\mu 0}^{2*}(\alpha, \beta, 0) \end{aligned} \right\} \quad (16)$$

and

$$\left. \begin{aligned} k_{\lambda} &= \frac{1}{\hbar} \langle n_1 K_1 | \int_{-\infty}^{\infty} \mathfrak{S}_1(t) dt | n_0 K_0 \rangle \\ &= \frac{4\pi Z_1 e}{(2\lambda + 1)\hbar} \langle n_1 K_1 | \mathfrak{M}_{\text{int}}^{*}(E\lambda, -K) | n_0 K_0 \rangle \\ &\quad \times \sum_{\mu} S_{\lambda\mu}(\vartheta, \xi) D_{\mu -K}^{\lambda*}(\alpha, \beta, 0). \end{aligned} \right\} \quad (17)$$

We have here introduced the notation $K = K_1 - K_0$ and

$$\xi = \frac{E_1 - E_0}{\hbar} \frac{a}{v}, \quad (18)$$

where a is half the distance of closest approach in a head-on collision, v is the relative velocity at large distances, and $E_1 - E_0$ is the energy difference between the intrinsic states $|n_1 K_1\rangle$ and $|n_0 K_0\rangle$. We shall evaluate the orbital integrals $S_{\lambda\mu}(\vartheta, \xi)$ in the coordinate system⁴⁾ where the z -axis is chosen along the symmetry axis of the hyperbolic orbit. In this coordinate system we introduce the functions $R_{\lambda\mu}(\vartheta, \xi)$ defined by the equation

$$R_{\lambda\mu}(\vartheta, \xi) = \frac{S_{\lambda\mu}(\vartheta, \xi)}{S_{\lambda 0}(\vartheta, 0)}. \quad (19)$$

With this notation, (16)–(17) may be written

$$k_0 = \frac{4}{3} q \sum_{\mu} R_{2\mu}(\vartheta, 0) D_{\mu 0}^{2*}(\alpha, \beta, 0) \quad (20)$$

$$k_{\lambda} = \chi^{(\lambda)} \sqrt{2\lambda + 1} \sum_{\mu} R_{\lambda\mu}(\vartheta, \xi) D_{\mu -K}^{\lambda*}(\alpha, \beta, 0) \quad (21)$$

in terms of the parameters q and $\chi^{(\lambda)}$ defined by⁴⁾

$$\left. \begin{aligned} q &= \frac{Z_1 e Q_0}{4 \hbar v a^2} \\ \chi^{(\lambda)} &= \frac{\sqrt{16 \pi} Z_1 e (\lambda - 1)!}{\hbar v a^{\lambda} (2\lambda + 1)!!} \langle n_0 K_0 | \mathfrak{M}_{\text{int}}(E\lambda, -K) | n_1 K_1 \rangle. \end{aligned} \right\} \quad (22)$$

The functions $R_{\lambda, \mu}(\vartheta, \xi)$, which directly determine the relative magnitude of the terms in (20)–(21), are defined in Appendix 1 in terms of the integrals $I_{\lambda, \mu}(\vartheta, \xi)$ (see ref. 6). For the case of $\lambda = 2$ and $\xi = 0$ they are tabulated in ref. 4. It is seen there that in the matrix element (20), where we have assumed $\xi = 0$, the $\mu = 0$ term dominates strongly over the $\mu \neq 0$ terms for all angles. In the following we therefore neglect these terms, i.e. we use

$$k_0 = \frac{4}{3} q R_{20}(\vartheta, 0) D_{00}^2(\alpha, \beta, 0) = \frac{4}{3} q_0(\vartheta) D_{00}^2(\alpha, \beta, 0). \quad (23)$$

For the second matrix element (21) we may use a similar approximation if ξ is small or if the scattering angle is close to 180 degrees. If the value of ξ is of the order of 0.1 or larger, the terms with $\mu \neq 0$ in (21) may contribute significantly, especially for forward scattering angles. We therefore include these terms in the computation of the excitation amplitude. Including also the possibility of mixed multipole excitation we find by expanding the product of D -functions on D -functions the following expression for the excitation amplitude (15)

$$b_{I_f M_f K_f, K_0} = -i \sum_{\lambda\mu} \chi^{(\lambda)} R_{\lambda\mu}(\vartheta, \xi) \langle I_f K_f M_f | C_{\lambda\mu}(q_0(\vartheta)) | I_i K_0 M_i \rangle \quad (24)$$

where

$$\left. \begin{aligned} & \langle I_f K_1 M_f | C_{\lambda\mu}(x) | I_i K_0 M_i \rangle \\ & = (-1)^{M_i - K_0} \sqrt{(2I_f + 1)(2I_i + 1)} \sum_I \sqrt{2I + 1} \begin{pmatrix} I_f & I_i & I \\ -M_f & M_i & \mu \end{pmatrix} \begin{pmatrix} I_f & I_i & I \\ -K_1 & K_0 & K \end{pmatrix} B_{IK}^{\lambda\mu}(x). \end{aligned} \right\} \quad (25)$$

The functions $B_{IK}^{\lambda\mu}(x)$ are given by

$$B_{IK}^{\lambda\mu}(x) = (-1)^{K-\mu} \sqrt{(2\lambda + 1)(2I + 1)} \sum_J (2J + 1) \begin{pmatrix} I & \lambda & J \\ -\mu & \mu & 0 \end{pmatrix} \begin{pmatrix} I & \lambda & J \\ -K & K & 0 \end{pmatrix} A_J(x) \quad (26)$$

in terms of the functions

$$A_J(x) = \frac{1}{2} \int_{-1}^1 P_J(y) e^{-i \frac{4}{3} x P_2(y)} dy \quad (27)$$

which are tabulated in refs. 4 and 8.

It is seen from eq. (25) that the dependence of the excitation amplitude on the quadrupole moment Q_0 is contained in the functions $B_{IK}^{\lambda\mu}$. Explicit formulae for these functions are given in Appendix 2 for $\lambda = 1, 2$ and 3, together with tables for the lowest values of I .

It is noted that even in the pure rotational model the intrinsic quadrupole moment may be different for different rotational bands. The series expansion (10) with ξ_0 given by (11) is, however, valid only for states where the quadrupole moments are nearly equal to Q_0 . A possible small deviation is contained in our expansion, and it is seen that to first order it gives no contribution to the excitation of the coupled band. To minimize the error one should, however, choose Q_0 to be the average quadrupole moment of the two bands.

The physical significance of the functions $B_{IK}^{\lambda\mu}(q)$ is illustrated by the special case of the excitation of an even-even nucleus with $I_i = K_0 = 0$. In this case one finds from (25)

$$\langle I_f K_1 M_f | C_{\lambda\mu}(q) | 000 \rangle = B_{I_f K_1}^{\lambda\mu}(q) \delta_{M_f \mu} (-1)^{\mu + K_1} \quad (28)$$

and therefore

$$b_{I_f M_f K_1 0} = -i \sum_{\lambda} \chi^{(\lambda)} R_{\lambda M_f}(\vartheta, \xi) (-1)^{M_f + K_1} B_{I_f K_1}^{\lambda M_f}. \quad (29)$$

Eq. (29) shows that the functions $B_{IK_1}^{\lambda\mu}$ for fixed λ, μ and K_1 are the relative excitation amplitudes for $E\lambda$ excitations of the states of spin I and magnetic quantum number $M_f = \mu$ in a band with $K = K_1$.

A comparison with Eq. 5.8 of ref. 4 shows that the function $B_{IK_1}^{\lambda\mu}$ is identical to the multiple quadrupole excitation amplitude $a_{I\mu}$ from a ground state $|\lambda K_1 \mu\rangle |K_1\rangle$ of spin λ and magnetic quantum number μ to a state $|IK_1 \mu\rangle |K_1\rangle$ in the ground state band which has $K = K_1$. From this identity follows immediately the relation

$$\sum_I |B_{IK}^{\lambda\mu}(q)|^2 = 1, \quad (30)$$

which simply expresses the fact that the nucleus in this model must stay in the ground state rotational band. By choosing the initial state to be a superposition of the states $|\lambda K_1 \mu\rangle |K_1\rangle$ and $|\lambda' K_1 \mu\rangle |K_1\rangle$ one may in a similar way show the more general relation

$$\sum_I (B_{IK}^{\lambda'\mu}(q))^* B_{IK}^{\lambda\mu}(q) = \delta_{\lambda\lambda'}. \quad (31)$$

From the excitation amplitude (24)–(27) one may compute the excitation probability $P_{I_f K_1}$ of the state with spin I_f in the $K = K_1$ band. One finds

$$\left. \begin{aligned} P_{I_f K_1} &= \frac{1}{2I_i + 1} \sum_{M_i M_f} |b_{I_f M_f K_1 K_0}|^2 \\ &= \sum_{\lambda\lambda'\mu} \chi^{(\lambda')} \chi^{(\lambda)} R_{\lambda'\mu}(\vartheta, \xi) R_{\lambda\mu}(\vartheta, \xi) (2I_f + 1) \\ &\quad \times \sum_I \begin{pmatrix} I_f & I_i & I \\ -K_1 & K_0 & K \end{pmatrix}^2 (B_{IK}^{\lambda'\mu}(q_0(\vartheta)))^* B_{IK}^{\lambda\mu}(q_0(\vartheta)). \end{aligned} \right\} \quad (32)$$

It is interesting also to compute the total excitation probability P_{K_1} of the $K = K_1$ band. From (32) one finds

$$P_{K_1} \equiv \sum_{I_f} P_{I_f K_1} = \sum_{\lambda} (\chi^{(\lambda)})^2 \sum_{\mu} (R_{\lambda\mu}(\vartheta, \xi))^2. \quad (33)$$

We have here used the completeness relation for the 3- j symbols and the orthogonality property (31) for the functions $B_{IK}^{\lambda\mu}(q)$. This result shows that the total excitation probability of the K_1 band is exactly the same as that one would find in the ordinary first order perturbation treatment. The effect of the multiple excitation that takes place is merely a redistribution of this probability on the various members of the band.

In the following we neglect the complications associated with the possibility of mixed multipole transitions between the two bands. For pure $E\lambda$ transitions the excitation probability (32) takes the simpler form

$$P_{I_f K_1} = (\chi^{(\lambda)})^2 \sum_{\mu} |R_{\lambda\mu}(\vartheta, \xi)|^2 (2I_f + 1) \sum_I \begin{pmatrix} I_f & I_i & I \\ -K_1 & K_0 & K \end{pmatrix}^2 |B_{IK}^{\lambda\mu}(q_0(\vartheta))|^2. \quad (34)$$

The functions $|B_{IK}^{\lambda\mu}|^2$ which determine the excitation probabilities are illustrated on Figs. 1 and 2 for the case of $\lambda = 2$. For $\lambda = 1, 2, 3$ and $\mu = 0$ they are given in Table 4, while for $\lambda = 2$ and $\mu \neq 0$ they are given in Table 5.

A further simplification in (34) is achieved for backward scatterings since $R_{\lambda\mu}(\pi, \xi)$ vanishes for $\mu \neq 0$. Also for other scattering angles one may, to a first approximation, neglect the terms in (34) with $\mu \neq 0$. In general, this $\mu = 0$ approximation is only accurate for large scattering angles. However, for small values of ξ the accuracy is quite good even for intermediate angles.

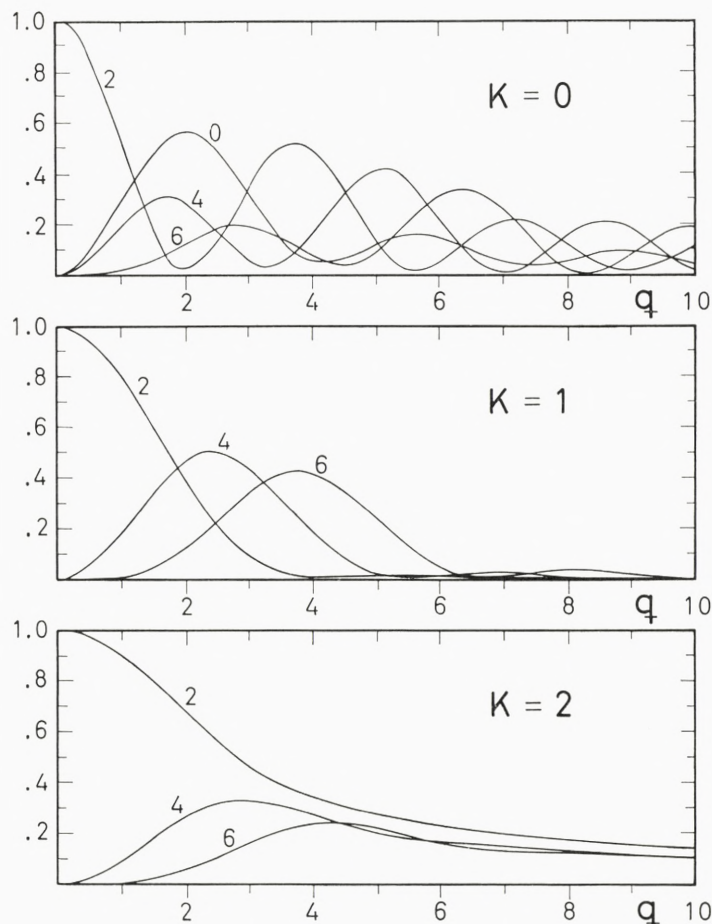


Fig. 1. The quantity $|B_{IK}^{\lambda}(q)|^2$ for $\lambda=2$ and $I \leq 6$ as a function of q . For backward scattering, these quantities give the relative quadrupole excitation probabilities in an even-even nucleus of the various rotational states in a band with $K = 0, 1$ or 2 . For other deflection angles they give the relative excitation probabilities of the magnetic substates with $M_j = 0$. The functions are given separately for the three values of $|K|$ while the spins I are indicated on the curves.

In the $\mu = 0$ approximation the excitation is determined by the quantities $B_{IK}^{\lambda 0} = B_{IK}^{\lambda}$. As can be seen from the definition (26), these functions vanish when $\lambda + I$ is odd. This is a reflection of the general selection rule that only natural parity states can be excited in even-even nuclei by backward scattering. The functions B_{IK}^{λ} are given in Table 3 for $\lambda = 1, 2$ and 3 .

As an example of the application of the results of this chapter we consider the quadrupole excitation of a band with $K = 2$ in an even-even nucleus. In this case, the excitation probability (34) takes the simple form

$$P_{I_f 2} = |\chi^{(2)}|^2 \sum_{\mu} |R_{2\mu}(\vartheta, \xi)|^2 |B_{I_f 2}^{\lambda \mu}(q_0(\vartheta))|^2. \quad (35)$$

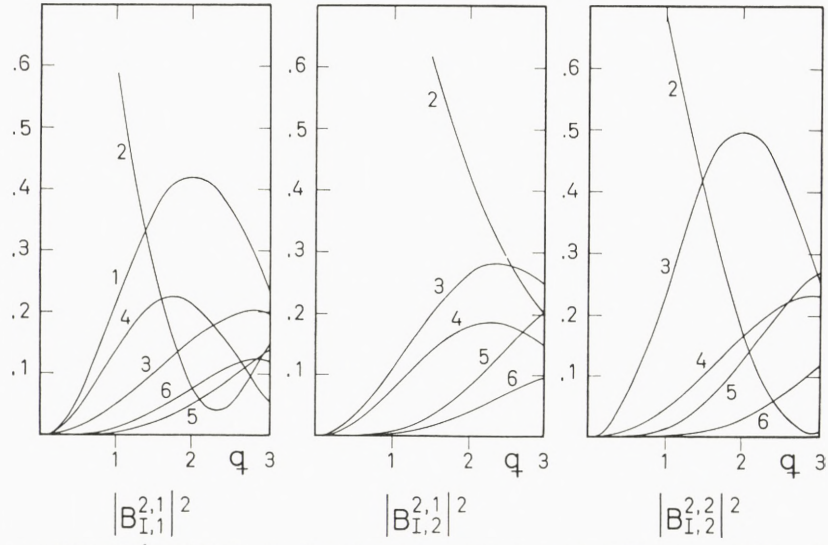


Fig. 2. The quantities $|B_{I,K}^{\lambda,\mu}(q)|^2$ for $\lambda=2$ and $I \leq 6$ as functions of q . These quantities give the relative quadrupole excitation probabilities in an even-even nucleus of the various rotational substates with spin I and fixed magnetic quantum number $M = \mu$ in a band with a definite value of K . The functions are given separately for different values of K and μ . Others may be obtained by means of the relations (67)–(68). The spin I is indicated on the curves.

The differential cross section which is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{I_f} = \frac{1}{4} a^2 \sin^{-4} \vartheta / 2 P_{I_f 2} \quad (36)$$

is illustrated in Fig. 3 as a function of ϑ for a special choice of parameters ξ , q and $\chi^{(2)}$ described in the figure caption. The figure also shows (by dashed curves) the result of the above mentioned $\mu = 0$ approximation where all terms in (35) with $\mu \neq 0$ are neglected. It is seen that this approximation is accurate only for rather large deflection angles.

For the excitation probabilities a considerable improvement over the $\mu = 0$ approximation can be achieved by assuming that the coefficients $B_{IK}^{\lambda,\mu}$ in (34) are independent of μ and equal to B_{IK}^{λ} . The contribution from the terms with $\mu \neq 0$ is then correct for forward scattering where $q_0(\vartheta) \ll 1$, since $B_{IK}^{\lambda,\mu}(0) = B_{IK}^{\lambda} = \delta_{I\lambda}$. In this approximation the excitation probabilities are given by

$$P_{I_f K_1} = [\chi^{(\lambda)}(\vartheta, \xi)]^2 (2I_f + 1) \sum_I \begin{pmatrix} I_f & I_i & I \\ -K_1 & K_0 & K \end{pmatrix}^2 |B_{IK}^{\lambda}(q_0(\vartheta))|^2 \quad (37)$$

with (cf. ref. 4)

$$\chi^{(\lambda)}(\vartheta, \xi) = \chi^{(\lambda)} \left(\sum_{\mu} [R_{\lambda\mu}(\vartheta, \xi)]^2 \right)^{1/2}. \quad (38)$$

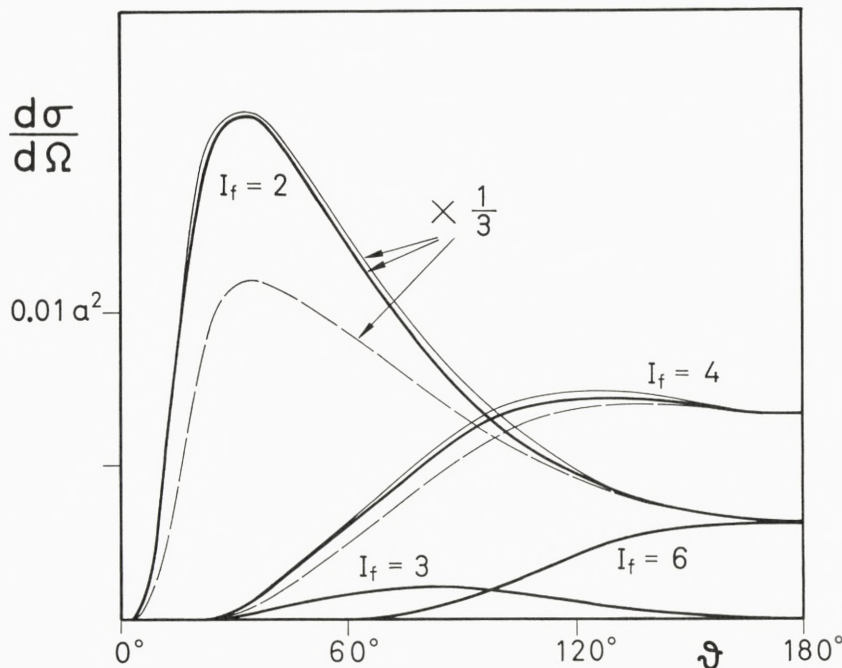


Fig. 3. The differential cross sections $d\sigma/d\Omega$ for the quadrupole Coulomb excitation of a band with $K = 2$ in an even-even nucleus. We have assumed that $q = 3$, $\chi^{(2)} = 0.4$ and $\xi = 0.3$, and the cross section is given in units of a^2 . The broad full-drawn curves show the result of applying Eqs. (35)–(36). The dashed curves and the thin full-drawn curves show the results of the $\mu = 0$ approximation and of the $\chi^{(\lambda)}(\vartheta, \xi)$ approximation, respectively. For the state with $I_f = 2$, which can be reached directly from the ground state the curves have been scaled down by a factor of 3. For the $I_f = 6$ state the three curves coincide within the accuracy of the drawing. For the $I_f = 3$ state no excitation takes place in the $\mu = 0$ and in the $\chi^{(2)}(\vartheta, \xi)$ approximations.

The approximation (37) is thus a modification of the $\mu = 0$ approximation where one applies the parameter $\chi^{(\lambda)}(\vartheta, \xi)$ instead of $\chi^{(\lambda)} R_{\lambda 0}(\vartheta, \xi)$. For backward scattering angles the two approximations coincide since $\chi^{(\lambda)}(\pi, \xi) = \chi^{(\lambda)} R_{\lambda 0}(\pi, \xi)$. For forward scattering angles, however, the expression (37) agrees with the result of the perturbation treatment, which applies in the limit of small ϑ . Finally, one obtains with (37) for all angles the correct result (33) for the total excitation probability of the K_1 -band.

The $\chi^{(\lambda)}(\vartheta, \xi)$ approximation is analogous to the $q(\vartheta)$ approximation of ref. 4; however, the systematic substitution of $q_0(\vartheta) = q_{\text{eff}}(\vartheta)$ with $q(\vartheta)$ is not expected to improve the approximation further in the present case.

For the special case which is illustrated in Fig. 3 the results of the $\chi^{(\lambda)}(\vartheta, \xi)$ approximation are indicated by thin full-drawn curves. A considerable improvement over the $\mu = 0$ approximation is noted for forward scatterings. For intermediate angles the expression (37) offers no significant improvement and in both approximations the excitation of the states of odd spins is neglected.

The accuracy of the $\chi^{(\lambda)}(\vartheta, \xi)$ approximation is expected to be inferior for larger values of q and ξ . This is connected with the fact that the terms with $\mu = 1$ in (34)

can be of importance already for angles where $q_0(\vartheta)$ is still so large that the coefficient $B_{IK}^{\lambda 1}$ is essentially different from B_{IK}^{λ} .

The functions $\chi^{(\lambda)}(\vartheta, \xi)$ are given for $\lambda = 1, 2$ and 3 in Table 2.

We conclude the present chapter by considering the effects which are associated with the fact that the nuclear wave functions are given by the symmetrized expressions (5) instead of the simple product (4).

For $K_0 = K_1 = 0$, where both the initial and final states are described by product wave functions, the Coulomb excitation amplitude of the state $|I_f K_1 M_f n_1\rangle$ is given by (24)

$$a_{I_f M} = b_{I_f M_f 00}. \quad (39)$$

It is noticed that the excitation amplitude $b_{I_f M_f 00}$, as seen from (25) and (26), vanishes unless $I_f + I_i + \lambda$ is even. Since on the other hand the symmetrization implies that in the K_1 -band only states of even spin or only states of odd spin occur, we obtain the following selection rule for the $K_0 = K_1 = 0$ case:

$$(-1)^{I_i + I_f} = \Delta\pi \quad (40)$$

where $\Delta\pi$ is the relative parity of the two bands.

If $K_0 = 0$ and $K_1 > 0$ (or $K_0 > 0$ and $K_1 = 0$) we find that the excitation amplitude of the state $|I_f K_1 M_f n_1\rangle$ is given by

$$a_{I_f M_f} = \frac{1}{\sqrt{2}} [b_{I_f M_f K_1 0} + (-1)^{I_i + K_1} b_{I_f M_f -K_1 0}]. \quad (41)$$

However, due to symmetry relations between matrix elements of the multipole moments the two terms of (41) are equal, and we get the result

$$a_{I_f M_f} = \sqrt{2} b_{I_f M_f K_1 0}. \quad (42)$$

This factor $\sqrt{2}$ will appear also in the ordinary perturbation treatment, and we thus still have the rule that the total excitation probability of the coupled band is given by the result of the perturbation calculation.

For the case where both $K_0 > 0$ and $K_1 > 0$ the excitation amplitude contains four terms. Due to symmetry relations the expression reduces to

$$a_{I_f M_f} = b_{I_f M_f K_1 K_0} + (-1)^{I_i + K_0} b_{I_f M_f K_1 -K_0}. \quad (43)$$

The two terms are here essentially different. The first term is proportional to the matrix element connecting the intrinsic states $|K_1\rangle$ and $|K_0\rangle$ while the second is proportional to the matrix element between $|K_1\rangle$ and $|-K_0\rangle$. If $K_0 + K_1 \leq \lambda$ both terms may contribute to the same transition.

IV. The Excitation of Mixed Bands

While, in the preceding chapter, we considered the Coulomb excitation of pure rotational bands, we shall now investigate the effect of deviations from the pure rotational model. These deviations are caused by the presence of the coupling term (6) in the nuclear Hamiltonian, which gives rise to admixtures in the pure rotational wave functions of states with different intrinsic structure.

We assume that the admixture is so small that it may be evaluated in a perturbation treatment (7)–(8). Even small mixings may, however, in some cases have an important effect on the Coulomb excitation amplitude. This is, e.g., the case if the excited band contains an admixture of the intrinsic wave function of the ground-state band. The contribution to the Coulomb excitation amplitude from the admixture is proportional to the large matrix elements between identical intrinsic states, and even when the admixture is small this contribution may be comparable to the contribution from the main part of the wave functions.^{(7) (11)}

Also mixing in the ground-state band or in the excited band of states with K values between K_0 and K_1 may be of importance. As an example one may consider the excitation of a band with $K_1 = K_0 + 3$, and with the same parity as the ground state. In this case, the direct transition is of $E4$ type. The mixing between the bands occurs only through a third-order term in H_c , and it may be expected that the admixture in the ground state band of states with $K = K_0 + 1$ as well as admixtures in the excited band with $K = K_0 + 2$ may lead to $E2$ transitions of comparable magnitude

In the following we limit ourselves to the case where the $E2$ transition between the bands is allowed. In this case the main correction to the results of Chapter III arises from the mixing between the two bands.

In Chapter II, the effect of the mixing was expressed through a modification (7)–(8) of the wave functions. An equivalent formulation of the mixing, which is more convenient for our purpose, is to maintain the pure rotational wave functions and instead modify the multipole operators.

When, in this representation, we expand the expression (1) for the Coulomb excitation amplitude, the only contribution to the excitation arises from the second term in (10). In this matrix element the operator $\mathfrak{S}_1(t)$ is given by the expressions (9), (12), and (13).

The multipole moment is, however, no longer given by the simple form (14) but may instead be expressed as

$$\left. \begin{aligned} \mathfrak{M}(E 2 \mu) &= e^{-iS} \sum_{\nu} D_{\mu\nu}^2(\alpha, \beta, 0) \mathfrak{M}_{\text{int}}(E 2 \nu) e^{iS} \\ &\approx \sum_{\nu} D_{\mu\nu}^2 \mathfrak{M}_{\text{int}}(E 2, \nu) - i \left[S, \sum_{\nu} D_{\mu\nu}^2 \mathfrak{M}_{\text{int}}(E 2 \nu) \right] \end{aligned} \right\} \quad (44)$$

to first order in S , where S is given by (8).

For the evaluation of the excitation amplitude we distinguish between the three cases where $|K_0 - K_1| = \Delta K = 1, 2$ and 0 .

For $\Delta K = 1$ we find from (44), utilizing (8), that the intrinsic matrix elements of $\mathfrak{M}(E 2 \mu)$ are given by

$$\left. \begin{aligned} & \langle n' \pm (K+1) | \mathfrak{M}(E 2, \mu) | n \pm K \rangle \\ & = \langle n' \pm (K+1) | \mathfrak{M}_{\text{int}}(E 2 \pm 1) | n \pm K \rangle D_{\mu \pm 1}^2 \\ & \quad \pm \varepsilon_1 \sqrt{\frac{5}{16\pi}} Q_0 \sqrt{6} D_{\mu \pm 1}^2. \end{aligned} \right\} \quad (45)$$

We have here neglected the terms with $\nu \neq 0$ in the commutator in (44) and only included the matrix elements proportional to the intrinsic quadrupole moment which again is assumed to be the same in the two bands.

As is seen from the result (45), the effect of the mixing is merely a change of the intrinsic matrix element of the quadrupole moment, and the change in the Coulomb excitation amplitude can therefore be described by a renormalization of χ .

For $\Delta K = 0$ one finds similarly from (44) and (8) the modified matrix element

$$\left. \begin{aligned} & \langle n' \pm K | \mathfrak{M}(E 2, \mu) | n \pm K \rangle = \langle n' \pm K | \mathfrak{M}_{\text{int}}(E 2 0) | n \pm K \rangle D_{\mu 0}^2 \\ & \quad + \varepsilon_0 \sqrt{\frac{5}{16\pi}} Q_0 [I^2, D_{\mu 0}^2]. \end{aligned} \right\} \quad (46)$$

Inserting (46) in (10) one finds that the correction due to the mixing is a matrix element of similar structure to that encountered in the computation of multiple Coulomb excitation for finite ξ (cf. ref. 4). The result may be expressed as the following correction to the excitation amplitude (15) (or (24)–(26)):

$$\left. \begin{aligned} \delta b_{I_f M_f K_0 K_0} & = -i \frac{4}{3} q \varepsilon_0 \sum_{\mu} R_{2\mu}(\vartheta, \xi) \langle I_f K_0 M_f | e^{-ik_0} 6 D_{\mu 0}^{2*} | I_i K_0 M_i \rangle \\ & \quad + i \frac{4}{3} q \varepsilon_0 \sum_{\mu} R_{2\mu}(\vartheta, \xi) \langle I_f K_0 M_f | e^{-ik_0} \sqrt{6} (D_{\mu 1}^{2*} I_- + D_{\mu -1}^{2*} I_+) | I_i K_0 M_i \rangle \\ & \quad - \left(\frac{4}{3} q\right)^2 \varepsilon_0 \sum_{\mu_1 \mu_2} \frac{1}{2} (R_{2\mu_1}(\vartheta, 0) R_{2\mu_2}(\vartheta, \xi) + i G_{\mu_1 \mu_2}(\vartheta, 0, \xi)) \\ & \quad \times \langle I_f K_0 M_f | e^{-ik_0} 6 (D_{\mu_1 1}^{2*} D_{\mu_2 -1}^{2*} + D_{\mu_1 -1}^{2*} D_{\mu_2 1}^{2*}) | I_i K_0 M_i \rangle \end{aligned} \right\} \quad (47)$$

where k_0 is given by (16). The last matrix element is multiplied by a second order orbital integral which we have separated in real and imaginary parts⁶⁾ according to the definition

$$G_{\mu_1 \mu_2}(\vartheta, \xi_1, \xi_2) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{d\xi'}{\xi'} R_{2\mu_1}(\vartheta, \xi_1 + \xi') R_{2\mu_2}(\vartheta, \xi_2 - \xi') \quad (48)$$

where P denotes the principal part. The function $G_{\mu_1 \mu_2}(\vartheta, \xi_1, \xi_2)$ has been tabulated¹⁰⁾ for a wide range of ξ_1 and ξ_2 values. Values for $\xi_1 = 0$ and $\mu_1 = \mu_2 = 0$ are given in Table 1.

In the expression (47) the first term represents a renormalization of the χ for the transition between the bands by an amount

$$\delta\chi = \frac{8}{\sqrt{5}} q \varepsilon_0.$$

The second term in (47) vanishes for even even nuclei. In other cases it may easily be expressed in terms of the functions $B_{I_1}^{2\mu}$. In the third term it is convenient to expand the product of the two D -functions on D -functions. In this way the matrix element can be expressed in terms of the functions $B_{I_0}^{J\mu}$ with $J = 0, 2, 4$.

We shall here only consider the $\mu = 0$ approximation. In this approximation one may write the expression (47) in the form

$$\left. \begin{aligned} \delta b_{I_f M_f K_0} &= \frac{R_{20}(\vartheta, \xi)}{R_{20}(\vartheta, 0)} \varepsilon_0 \langle I_f K_0 M_f | [I^2, e^{-ik_0}] | I_i K_0 M_i \rangle \\ &- i 6 \left(\frac{4}{3} q\right)^2 \varepsilon_0 G_{00}(\vartheta, 0, \xi) \times \langle I_f K_0 M_f | e^{-ik_0} D_{01}^2 D_{0-1}^2 | I_i K_0 M_i \rangle \\ &= \frac{R_{20}(\vartheta, \xi)}{R_{20}(\vartheta, 0)} \varepsilon_0 [I_f(I_f+1) - I_i(I_i+1)] B_{I_f K_0}^{I_i M_i}(q_0(\vartheta)) \\ &- i 6 \left(\frac{4}{3} q\right)^2 \varepsilon_0 G_{00}(\vartheta, 0, \xi) \\ &\times \sum_J \sqrt{2J+1} \begin{pmatrix} 22J \\ 000 \end{pmatrix} \begin{pmatrix} 22J \\ -110 \end{pmatrix} \langle I_f K_0 M_f | C_{J0}(q_0(\vartheta)) | I_i K_0 M_i \rangle \end{aligned} \right\} \quad (49)$$

where the matrix elements of $C_{J0}(q)$ are given by (25).

For $\xi = 0$ the last term in (49) vanishes since $G_{00}(\vartheta, 0, 0) = 0$. The resultant simple expression for δb can in this case be obtained more directly inserting the mixed wave functions (7) in the expression (10). The correction term δb then arises from the first term in (10).

For small values of ξ one may still neglect the second term in (49). The order of magnitude of the error may be estimated by the number $q^2 \varepsilon_0 G_{00}(\vartheta, 0, \xi)$.

It is noted that the symmetrization of the rotational wave functions has no effect on the result (49) except for $K_0 = \frac{1}{2}$, where additional corrections with $\Delta K = 1$ may appear, and for $K_0 = 1$, where one obtains additional terms with $\Delta K = 2$.

TABLE 1.

The quantity $G_{00}(\vartheta, 0, \xi)$. This function which is defined by Eq. (48) may be used for the computation of quadrupole Coulomb excitation of impure bands for finite ξ . The entries are given in the form of a number followed by the power of ten, by which it should be multiplied.

$\vartheta \setminus \xi$	0.1	0.2	0.4	0.6	0.8	1.2	2.0
180°	1.530 (-1)	2.459 (-1)	3.031 (-1)	2.711 (-1)	2.114 (-1)	1.048 (-1)	1.729 (-2)
160°	1.442 (-1)	2.321 (-1)	2.870 (-1)	2.574 (-1)	2.013 (-1)	1.003 (-1)	1.668 (-2)
140°	1.203 (-1)	1.946 (-1)	2.430 (-1)	2.198 (-1)	1.732 (-1)	8.737 (-2)	1.485 (-2)
120°	8.776 (-2)	1.435 (-1)	1.820 (-1)	1.668 (-1)	1.329 (-1)	6.817 (-2)	1.183 (-2)
100°	5.479 (-2)	9.106 (-2)	1.184 (-1)	1.102 (-1)	8.871 (-2)	4.600 (-2)	7.972 (-3)
80°	2.812 (-2)	4.801 (-2)	6.434 (-2)	6.070 (-2)	4.900 (-2)	2.509 (-2)	4.090 (-3)
60°	1.101 (-2)	1.965 (-2)	2.714 (-2)	2.546 (-2)	2.008 (-2)	9.534 (-3)	1.272 (-3)
40°	2.831 (-3)	5.386 (-3)	7.354 (-3)	6.373 (-3)	4.514 (-3)	1.665 (-3)	1.261 (-4)
20°	3.153 (-4)	5.861 (-4)	5.722 (-4)	3.206 (-4)	1.416 (-4)	1.934 (-5)	1.855 (-7)

For $\Delta K = 2$ mixing the evaluation of the corrections to the excitation amplitude is completely analogous to the case of $\Delta K = 0$. We shall here only quote the result for the $\mu = 0$ approximation. One finds

$$\begin{aligned}
 \delta b_{I_f M_f K_0 \pm 2 K_0} &= \frac{R_{20}(\vartheta, \xi)}{R_{20}(\vartheta, 0)} \varepsilon_{\pm 2} \langle I_f K_0 \pm 2 M_f | [I_{\mp}^2, e^{-ik_0}] | I_i K_0 M_i \rangle \\
 &\quad - i \left(\frac{4}{3} q \right)^2 G_{00}(\vartheta, 0, \xi) \varepsilon_{\pm 2} \times \langle I_f K_0 \pm 2 M_f | e^{-ik_0} (D_{0\pm 1}^2)_i | I_0 K_0 M_i \rangle \\
 &= \frac{R_{20}(\vartheta, \xi)}{R_{20}(\vartheta, 0)} \varepsilon_{\pm 2} \left[\sqrt{(I_f \pm K_0 + 2)(I_f \pm K_0 + 1)(I_f \mp K_0)(I_f \mp K_0 - 1)} B_{I_f K_0}^{I_i M_i}(q_0(\vartheta)) \right. \\
 &\quad \left. - \sqrt{(I_i \pm K_0 + 2)(I_i \pm K_0 + 1)(I_i \mp K_0)(I_i \mp K_0 - 1)} B_{I_f K_0 \pm 2}^{I_i M_i}(q_0(\vartheta)) \right] \\
 &\quad - i 6 \left(\frac{4}{3} q \right)^2 \varepsilon_{\pm 2} G_{00}(\vartheta, 0, \xi) \sum_J (2J+1)^{1/2} \begin{pmatrix} 2 & 2 & J \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2 & 2 & J \\ 1 & 1 & -2 \end{pmatrix} \\
 &\quad \times \langle I_f K_0 \pm 2 M_f | C_{J0}(q_0(\vartheta)) | I_i K_0 M_i \rangle.
 \end{aligned} \tag{50}$$

Again the simple result for $\xi = 0$ can be obtained directly inserting the mixed wave functions (7) in (10). For $K_0 = 0$ the symmetrization gives rise to an additional factor $\sqrt{2}$.

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Appendix 1

In the present work the orbital integrals $S_{\lambda\mu}(\vartheta, \xi)$ are to be evaluated in the coordinate system, where the z axis is chosen in the plane of the orbit in such a way that it bisects the angle between $-\vec{v}_i$ and \vec{v}_f , where \vec{v}_i and \vec{v}_f are the initial and final velocities of the projectile. We shall here express the integrals $S_{\lambda\mu}(\vartheta, \xi)$ by functions $R_{\lambda\mu}(\vartheta, \xi)$ which are normalized in such a way that they attain the value 1 for $\mu = 0$, $\vartheta = \pi$ and $\xi = 0$, i. e.

$$S_{\lambda\mu}(\vartheta, \xi) = \frac{1}{v\alpha^\lambda} \sqrt{\frac{2\lambda+1}{\pi}} \frac{(\lambda-1)!}{(2\lambda-1)!!} R_{\lambda\mu}(\vartheta, \xi). \tag{51}$$

Since the orbital integrals transform like spherical tensors under rotations of the coordinate system, the functions $R_{\lambda\mu}(\vartheta, \xi)$ are related to the integrals $I_{\lambda\mu}(\vartheta, \xi)$ in the focal system (cf. refs. 5 and 8) by the relation

$$R_{\lambda\mu}(\vartheta, \xi) = \frac{(2\lambda-1)!!}{2(\lambda-1)!} \sum_{\nu} D_{\mu\nu}^\lambda\left(0, \frac{\pi}{2}, 0\right) D_{0\nu}^\lambda\left(0, \frac{\pi}{2}, 0\right) I_{\lambda\nu}(\vartheta, \xi), \tag{52}$$

where we have utilized the following explicit formula for $I_{\lambda\mu}(\pi, 0)$:

$$I_{\lambda\mu}(\pi, 0) = 2 \frac{(\lambda-1)!}{(2\lambda-1)!!}. \tag{53}$$

The functions $R_{\lambda\mu}(\vartheta, \xi)$ satisfy the following symmetry relation

$$R_{\lambda-\mu}(\vartheta, \xi) = R_{\lambda\mu}(\vartheta, \xi). \tag{54}$$

Furthermore one finds

$$R_{\lambda\mu}(\vartheta, -\xi) = (-1)^\mu R_{\lambda\mu}(\vartheta, \xi), \tag{55}$$

which implies that $R_{\lambda\mu}(\vartheta, 0)$ vanishes for odd values of μ . Finally we note the following property:

$$R_{\lambda\mu}(\pi, \xi) = 0 \quad \text{for} \quad \mu \neq 0. \tag{56}$$

A tabulation of the functions $R_{\lambda\mu}(\vartheta, \xi)$ will be published separately¹⁰⁾. For $\xi = 0$ the functions $R_{\lambda\mu}(\vartheta, 0)$ are proportional to the functions $J_{\lambda\mu}(\vartheta)$ which for $\lambda = 2$ are tabulated in ref. 4. The relations are

$$\left. \begin{aligned} R_{2,0}(\vartheta, 0) &= \frac{3}{4} J_{2,0}(\vartheta) \\ R_{2,2}(\vartheta, 0) &= -\frac{3\sqrt{3}}{4\sqrt{2}} J_{2,2}(\vartheta). \end{aligned} \right\} \tag{57}$$

TABLE 2.

The quantity $\chi^{(\lambda)}(\vartheta, \xi)/\chi^{(\lambda)}$ for $\lambda = 1, 2$ and 3 as a function of ϑ and ξ . The square of this function gives the variation of the total excitation probability of the intrinsic state with particle energy and deflection angle, relative to the case of $\vartheta = \pi$ and $\xi = 0$. It is noted that $\chi^{(\lambda)}(\pi, \xi)/\chi^{(\lambda)} = R_{\lambda 0}(\pi, \xi)$. Furthermore, $\chi^{(\lambda)}(\vartheta, 0)/\chi^{(\lambda)} \approx R_{\lambda 0}(\vartheta, 0)$ which implies that $q_0(\vartheta)/q \approx \chi^{(2)}(\vartheta, 0)/\chi^{(2)}$.

 $\chi^{(1)}(\vartheta, \xi)$

$\vartheta \backslash \xi$	0.0	0.2	0.4	0.6	0.8	1.0	2.0
180°	1.0000	0.6344	0.3734	0.2140	0.1209	0.06766	0.003449
160°	0.9848	0.6261	0.3695	0.2123	0.1202	0.06740	0.003467
140°	0.9397	0.6014	0.3573	0.2065	0.1175	0.06618	0.003455
120°	0.8660	0.5601	0.3356	0.1950	0.1114	0.06283	0.003261
100°	0.7660	0.5017	0.3022	0.1755	0.0997	0.05582	0.002736
80°	0.6428	0.4261	0.2547	0.1453	0.0807	0.04398	0.001847
60°	0.5000	0.3320	0.1908	0.1030	0.0537	0.02745	0.000815
40°	0.3420	0.2179	0.1100	0.0511	0.0228	0.00990	0.000128
20°	0.1736	0.0828	0.0252	0.0069	0.0018	0.00045	0.000000

 $\chi^{(2)}(\vartheta, \xi)$

$\vartheta \backslash \xi$	0.0	0.2	0.4	0.6	0.8	1.0	2.0
180°	1.0000	0.8342	0.5929	0.3924	0.2488	0.1536	0.01100
160°	0.9728	0.8148	0.5828	0.3882	0.2479	0.1539	0.01133
140°	0.8943	0.7579	0.5519	0.3742	0.2428	0.1530	0.01191
120°	0.7729	0.6679	0.4992	0.3461	0.2288	0.1462	0.01182
100°	0.6216	0.5513	0.4243	0.3000	0.2006	0.1290	0.01026
80°	0.4561	0.4175	0.3294	0.2345	0.1560	0.0991	0.00704
60°	0.2932	0.2779	0.2204	0.1531	0.0978	0.0591	0.00310
40°	0.1498	0.1462	0.1095	0.0682	0.0382	0.0200	0.00048
20°	0.0438	0.0407	0.0209	0.0081	0.0027	0.0008	0.00000

 $\chi^{(3)}(\vartheta, \xi)$

$\vartheta \backslash \xi$	0.0	0.2	0.4	0.6	0.8	1.0	2.0
180°	1.0000	0.9077	0.7201	0.5268	0.3653	0.2439	0.02345
160°	0.9624	0.8761	0.7008	0.5176	0.3626	0.2445	0.02461
140°	0.8528	0.7862	0.6436	0.4878	0.3503	0.2419	0.02671
120°	0.6935	0.6513	0.5518	0.4327	0.3203	0.2268	0.02707
100°	0.5102	0.4908	0.4327	0.3512	0.2667	0.1923	0.02353
80°	0.3307	0.3269	0.2998	0.2497	0.1920	0.1387	0.01584
60°	0.1778	0.1813	0.1715	0.1434	0.1083	0.0758	0.00672
40°	0.0692	0.0725	0.0687	0.0539	0.0370	0.0230	0.00099
20°	0.0118	0.0128	0.0098	0.0052	0.0009	0.0009	0.00000

It was shown in Chapter III that in many cases it will be sufficient to know the following combination of the functions $R_{\lambda\mu}(\vartheta, \xi)$:

$$\chi^{(\lambda)}(\vartheta, \xi)/\chi^{(\lambda)} = \left[\sum_{\mu} (R_{\lambda\mu}(\vartheta, \xi))^2 \right]^{1/2}. \quad (58)$$

This function is related to the differential excitation function $df_{E\lambda}(\vartheta, \xi)$ in the first order perturbation theory (cf. refs. 5 and 8) by the formula

$$\chi^{(\lambda)}(\vartheta, \xi)/\chi^{(\lambda)} = [df_{E\lambda}(\vartheta, \xi)/df_{E\lambda}(\pi, 0)]^{1/2} \sin^2 \vartheta/2. \quad (59)$$

These functions are given in Table 2 for $\lambda = 1, 2$ and 3.

Appendix 2

The excitation amplitudes of states in deformed nuclei are all expressed through the functions $B_{IK}^{\lambda\mu}$ which may be written in the form

$$B_{IK}^{\lambda\mu}(q) = \langle IK\mu | \sqrt{2\lambda+1} D_{\mu K}^{\lambda} e^{-i\frac{4}{3}qD_{30}^{\lambda}} | 000 \rangle. \quad (60)$$

In the $\chi^{(\lambda)}(\vartheta, \xi)$ approximation and in the $\mu = 0$ approximation one needs only the functions (60) with $\mu = 0$. For these quantities we use the notation B_{IK}^{λ} , i.e.

$$B_{IK}^{\lambda 0} = B_{IK}^{\lambda}. \quad (61)$$

From Eq. (26) one finds for $\lambda = 1, 2$ and 3 the following explicit expressions for B_{IK}^{λ} in terms of the functions A_I defined by (27):

$$\left. \begin{aligned} B_{I,0}^1 &= \sqrt{\frac{3}{2I+1}} [IA_{I-1} + (I+1)A_{I+1}] \\ B_{I,1}^1 &= \sqrt{\frac{3I(I+1)}{2(2I+1)}} [A_{I-1} - A_{I+1}] \end{aligned} \right\} \quad (62)$$

$$\left. \begin{aligned} B_{I,0}^2 &= \sqrt{\frac{5(2I+1)}{4}} \left[\frac{3(I-1)I}{(2I-1)(2I+1)} A_{I-2} + \frac{2I(I+1)}{(2I-1)(2I+3)} A_I \right. \\ &\quad \left. + \frac{3(I+1)(I+2)}{(2I+1)(2I+3)} A_{I+2} \right] \\ B_{I,1}^2 &= \sqrt{\frac{15I(I+1)(2I+1)}{2}} \left[\frac{I-1}{(2I-1)(2I+1)} A_{I-2} \right. \\ &\quad \left. + \frac{1}{(2I-1)(2I+3)} A_I - \frac{I+2}{(2I+1)(2I+3)} A_{I+2} \right] \\ B_{I,2}^2 &= \sqrt{\frac{15(I-1)I(I+1)(I+2)(2I+1)}{8}} \left[\frac{1}{(2I-1)(2I+1)} A_{I-2} \right. \\ &\quad \left. - \frac{2}{(2I-1)(2I+3)} A_I + \frac{1}{(2I+1)(2I+3)} A_{I+2} \right] \end{aligned} \right\} \quad (63)$$

$$\left. \begin{aligned}
B_{I,0}^3 &= \sqrt{\frac{7}{4(2I+1)}} \left[\frac{5(I-2)(I-1)I}{(2I-3)(2I-1)} A_{I-3} + \frac{3(I-1)I(I+1)}{(2I-3)(2I+3)} A_{I-1} \right. \\
&\quad \left. + \frac{3I(I+1)(I+2)}{(2I-1)(2I+5)} A_{I+1} + \frac{5(I+1)(I+2)(I+3)}{(2I+3)(2I+5)} A_{I+3} \right] \\
B_{I,1}^3 &= \sqrt{\frac{21I(I+1)}{16(2I+1)}} \left[\frac{5(I-2)(I-1)}{(2I-3)(2I-1)} A_{I-3} + \frac{(I-1)(I+6)}{(2I-3)(2I+3)} A_{I-1} \right. \\
&\quad \left. - \frac{(I-5)(I+2)}{(2I-1)(2I+5)} A_{I+1} - \frac{5(I+2)(I+3)}{(2I+3)(2I+5)} A_{I+3} \right] \\
B_{I,2}^3 &= \sqrt{\frac{105(I-1)I(I+1)(I+2)}{8(2I+1)}} \left[\frac{I-2}{(2I-3)(2I-1)} A_{I-3} \right. \\
&\quad \left. - \frac{I-3}{(2I-3)(2I+3)} A_{I-1} - \frac{I+4}{(2I-1)(2I+5)} A_{I+1} + \frac{I+3}{(2I+3)(2I+5)} A_{I+3} \right] \\
B_{I,3}^3 &= \sqrt{\frac{35(I-2)(I-1)I(I+1)(I+2)(I+3)}{16(2I+1)}} \left[\frac{1}{(2I-3)(2I-1)} A_{I-3} \right. \\
&\quad \left. - \frac{3}{(2I-3)(2I+3)} A_{I-1} + \frac{3}{(2I-1)(2I+5)} A_{I+1} - \frac{1}{(2I+3)(2I+5)} A_{I+3} \right].
\end{aligned} \right\} (64)$$

TABLE 3.

The quantities $B_{IK}^\lambda(q)$. The table lists the real and imaginary part of this quantity for $\lambda = 1, 2$ and 3 , and $I < \lambda + 4$ as a function of q . In the use of the tables the symmetry relations (65) should be employed. The quantity B_{00}^2 has been omitted because of its simple relation (66) to the function $A_2(q)$. The tables have been computed by means of the relations (62)–(64) from the functions $A_I(q)$ as they are tabulated in ref. 4.

q	Re $B_{1,0}^1$	Im $B_{1,0}^1$	Re $B_{3,0}^1$	Im $B_{3,0}^1$	Re $B_{5,0}^1$	Im $B_{5,0}^1$
0.0	1.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.5	0.93169	-0.25556	-0.05647	-0.24974	-0.03190	0.00664
1.0	0.74457	-0.44841	-0.20654	-0.43136	-0.11374	0.04960
1.5	0.48634	-0.53440	-0.39849	-0.49869	-0.20901	0.14865
2.0	0.22007	-0.50052	-0.56619	-0.44190	-0.27331	0.29737
2.5	0.00530	-0.36721	-0.65117	-0.28935	-0.27188	0.46337
3.0	-0.11902	-0.10851	-0.62214	-0.09682	-0.19400	0.59947
3.5	-0.14402	0.00361	-0.48453	0.07219	-0.05789	0.66093
4.0	-0.09039	0.13687	-0.27727	0.16768	0.09494	0.62240
4.5	0.00148	0.19262	-0.05866	0.16881	0.21476	0.48819
5.0	0.02204	0.17150	0.11394	0.08861	0.26156	0.29196
5.5	0.13098	0.08816	0.20260	-0.04283	0.21978	0.08571
6.0	0.12080	0.00854	0.19958	-0.14671	0.10398	-0.07774
6.5	0.06502	-0.06088	0.12711	-0.20908	-0.04646	-0.16222
7.0	-0.01110	-0.08843	0.02663	-0.20029	-0.18163	-0.15950
7.5	-0.07736	-0.07127	-0.05772	-0.12758	-0.25860	-0.09024
8.0	-0.11025	-0.02414	-0.09487	-0.02083	-0.25622	0.00527
8.5	-0.10116	0.02851	-0.07653	0.08404	-0.18124	0.08288
9.0	-0.04457	0.06324	-0.01760	0.14214	-0.06440	0.11078
9.5	-0.00080	0.06650	0.05204	0.14786	0.05192	0.07997
10.0	0.04761	0.03906	0.10086	0.10309	0.12973	0.00597

(continued)

TABLE 3 (continued).

q	Re $B_{1,1}^1$	Im $B_{1,1}^1$	Re $B_{3,1}^1$	Im $B_{3,1}^1$	Re $B_{5,1}^1$	Im $B_{5,1}^1$
0.0	1.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.5	0.96853	0.13192	0.00013	-0.20952	-0.02524	0.00127
1.0	0.87738	0.25542	0.00224	-0.39422	-0.09593	0.00963
1.5	0.73585	0.36224	0.01063	-0.53335	-0.19807	0.02908
2.0	0.55838	0.44466	0.03060	-0.61352	-0.31205	0.05849
2.5	0.36086	0.49598	0.06616	-0.63025	-0.41698	0.09136
3.0	0.16208	0.51135	0.11804	-0.58788	-0.49517	0.11731
3.5	-0.02177	0.48850	0.18236	-0.49778	-0.53542	0.12510
4.0	-0.17709	0.42853	0.25089	-0.37551	-0.53429	0.10606
4.5	-0.29408	0.33620	0.31239	-0.23768	-0.49568	0.05702
5.0	-0.36728	0.21883	0.35487	-0.09947	-0.42833	-0.01818
5.5	-0.39549	0.09542	0.36840	0.01944	-0.34303	-0.10924
6.0	-0.38122	-0.03880	0.34738	0.13412	-0.24989	-0.20145
6.5	-0.32911	-0.15550	0.29177	0.21654	-0.15631	-0.27909
7.0	-0.25014	-0.24824	0.20729	0.27214	-0.06664	-0.32881
7.5	-0.15095	-0.30866	0.10418	0.30019	0.01697	-0.34249
8.0	-0.04305	-0.33224	-0.00476	0.30096	0.09334	-0.31852
8.5	0.06282	-0.31871	-0.10652	0.27556	0.16063	-0.26152
9.0	0.15659	-0.27180	-0.19001	0.22636	0.21563	-0.18082
9.5	0.22951	-0.19852	-0.24730	0.15737	0.25367	-0.08792
10.0	0.27507	-0.10821	-0.27453	0.07463	0.26994	0.00560

q	Re $B_{2,0}^2$	Im $B_{2,0}^2$	Re $B_{4,0}^2$	Im $B_{4,0}^2$	Re $B_{6,0}^2$	Im $B_{6,0}^2$
0.0	1.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.5	0.90692	-0.17790	-0.04471	-0.24229	-0.03080	0.00548
1.0	0.65243	-0.28547	-0.15906	-0.41037	-0.10932	0.04098
1.5	0.30295	-0.27690	-0.29108	-0.45511	-0.19916	0.12000
2.0	-0.05430	-0.14681	-0.37667	-0.36841	-0.25659	0.23110
2.5	-0.33783	0.06798	-0.36414	-0.18469	-0.24903	0.33959
3.0	-0.49570	0.30096	-0.23477	0.03274	-0.16932	0.39971
3.5	-0.51794	0.47857	-0.01108	0.21456	-0.03934	0.37311
4.0	-0.43492	0.54458	0.24965	0.30997	0.09738	0.24576
4.5	-0.30242	0.47804	0.47471	0.30320	0.19253	0.03431
5.0	-0.17974	0.29879	0.60044	0.21656	0.21204	-0.20721
5.5	-0.10880	0.05215	0.59355	0.09038	0.15002	-0.42244
6.0	-0.10157	-0.17497	0.46128	0.00879	0.03191	-0.55300
6.5	-0.13943	-0.34505	0.24735	-0.01475	-0.09414	-0.56771
7.0	-0.18415	-0.41829	0.01557	0.03987	-0.17543	-0.47049
7.5	-0.19537	-0.39438	-0.17234	0.15016	-0.17438	-0.29825
8.0	-0.14741	-0.30135	-0.27675	0.26984	-0.08308	-0.10701
8.5	-0.03966	-0.18188	-0.29053	0.34728	0.07334	0.04828
9.0	0.10353	-0.07638	-0.23729	0.34539	0.24548	0.13222
9.5	0.24223	-0.00915	-0.15846	0.25495	0.37809	0.13952
10.0	0.33620	0.01757	-0.09524	0.09786	0.42535	0.09404

(continued)

TABLE 3 (continued).

q	$\text{Re } B_{2,1}^2$	$\text{Im } B_{2,1}^2$	$\text{Re } B_{4,1}^2$	$\text{Im } B_{4,1}^2$	$\text{Re } B_{6,1}^2$	$\text{Im } B_{6,1}^2$
0.0	1.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.5	0.96867	-0.09207	-0.02765	-0.22720	-0.02777	0.00377
1.0	0.87978	-0.16602	-0.10317	-0.41953	-0.10344	0.02816
1.5	0.74722	-0.20793	-0.20647	-0.54991	-0.20628	0.08670
2.0	0.59073	-0.21122	-0.31017	-0.60501	-0.30889	0.18043
2.5	0.43159	-0.17778	-0.38689	-0.58708	-0.38478	0.29681
3.0	0.28827	-0.11712	-0.41638	-0.51228	-0.41591	0.41334
3.5	0.17318	-0.04365	-0.39051	-0.40540	-0.39740	0.50391
4.0	0.09113	0.02710	-0.31448	-0.29294	-0.33800	0.54634
4.5	0.03988	0.08211	-0.20488	-0.19656	-0.25656	0.52891
5.0	0.01209	0.11349	-0.08438	-0.12855	-0.17570	0.45365
5.5	-0.00165	0.11620	0.02430	-0.09906	-0.11482	0.33554
6.0	-0.00986	0.10458	0.10385	-0.07545	-0.08464	0.19801
6.5	-0.01820	0.07600	0.14626	-0.07098	-0.08486	0.06656
7.0	-0.02854	0.04269	0.15280	-0.06508	-0.10541	-0.03802
7.5	-0.03957	0.01225	0.13236	-0.04953	-0.13066	-0.10416
8.0	-0.04814	-0.01050	0.09757	-0.02230	-0.14499	-0.13112
8.5	-0.05106	-0.02413	0.06062	0.01273	-0.13778	-0.12732
9.0	-0.04654	-0.02981	0.03008	0.04790	-0.10651	-0.10607
9.5	-0.03487	-0.03030	0.00946	0.07489	-0.05673	-0.08078
10.0	-0.01841	-0.02843	-0.00235	0.08760	0.00034	-0.06093

q	$\text{Re } B_{2,2}^2$	$\text{Im } B_{2,2}^2$	$\text{Re } B_{4,2}^2$	$\text{Im } B_{4,2}^2$	$\text{Re } B_{6,2}^2$	$\text{Im } B_{6,2}^2$
0.0	1.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.5	0.96849	0.18792	0.00992	-0.16214	-0.01772	-0.00004
1.0	0.87678	0.36078	0.03907	-0.30790	-0.06818	-0.00019
1.5	0.73301	0.50478	0.08530	-0.42281	-0.14366	-0.00120
2.0	0.54984	0.60862	0.14507	-0.49610	-0.23268	-0.00482
2.5	0.34317	0.66442	0.21336	-0.52180	-0.32198	-0.01345
3.0	0.13054	0.66846	0.28381	-0.49922	-0.39857	-0.03001
3.5	-0.07052	0.62154	0.34912	-0.43275	-0.45165	-0.05696
4.0	-0.24415	0.52888	0.40155	-0.33099	-0.47384	-0.09537
4.5	-0.37757	0.39972	0.43395	-0.20557	-0.46214	-0.14425
5.0	-0.46213	0.24642	0.44050	-0.06967	-0.41752	-0.19999
5.5	-0.49395	0.08331	0.41776	0.05752	-0.34462	-0.25683
6.0	-0.47406	-0.07464	0.36528	0.18190	-0.25066	-0.30731
6.5	-0.40809	-0.21337	0.28595	0.27569	-0.14417	-0.34360
7.0	-0.30555	-0.32097	0.18588	0.33794	-0.03410	-0.35881
7.5	-0.17879	-0.38889	0.07378	0.36492	0.07094	-0.34821
8.0	-0.04178	-0.41268	-0.04000	0.35618	0.16344	-0.31030
8.5	0.09130	-0.39236	-0.14472	0.31441	0.23709	-0.24700
9.0	0.20736	-0.33228	-0.23051	0.24501	0.28726	-0.16365
9.5	0.29560	-0.24056	-0.28954	0.15562	0.31083	-0.06803
10.0	0.34843	-0.12816	-0.31686	0.05539	0.30675	0.03046

(continued)

TABLE 3 (continued).

q	$\text{Re } B_{3,0}^3$	$\text{Im } B_{3,0}^3$	$\text{Re } B_{5,0}^3$	$\text{Im } B_{5,0}^3$	$\text{Re } B_{7,0}^3$	$\text{Im } B_{7,0}^3$
0.0	1.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.5	0.92002	-0.16436	-0.04240	-0.24144	-0.03065	0.00530
1.0	0.70468	-0.25402	-0.15033	-0.41653	-0.10975	0.03966
1.5	0.41948	-0.22136	-0.27340	-0.48338	-0.20401	0.11547
2.0	0.14795	-0.06293	-0.34988	-0.43871	-0.27459	0.22299
2.5	-0.03650	0.17913	-0.33166	-0.31801	-0.29269	0.32940
3.0	-0.09545	0.43210	-0.20395	-0.18144	-0.25409	0.39269
3.5	-0.03553	0.61829	0.00782	-0.08956	-0.18074	0.37852
4.0	0.09555	0.68187	0.24605	-0.08238	-0.11229	0.27631
4.5	0.22908	0.60745	0.44146	-0.15995	-0.08957	0.10550
5.0	0.30095	0.42378	0.53791	-0.28737	-0.13657	-0.08861
5.5	0.27476	0.18559	0.51238	-0.41581	-0.24866	-0.25072
6.0	0.15361	-0.01684	0.38241	-0.45984	-0.39192	-0.33508
6.5	-0.02306	-0.14678	0.19888	-0.41197	-0.51432	-0.32258
7.0	-0.19579	-0.17469	0.02641	-0.26452	-0.56498	-0.22795
7.5	-0.30786	-0.11523	-0.08031	-0.05527	-0.51357	-0.09405
8.0	-0.32584	-0.01197	-0.09569	0.15547	-0.36295	0.02404
8.5	-0.25046	0.08137	-0.03069	0.30708	-0.14934	0.08025
9.0	-0.11437	0.12212	0.07369	0.36005	0.06985	0.05437
9.5	0.03175	0.09365	0.16384	0.30917	0.23581	-0.04032
10.0	0.13884	0.00984	0.19512	0.18396	0.30896	-0.16299

q	$\text{Re } B_{3,1}^3$	$\text{Im } B_{3,1}^3$	$\text{Re } B_{5,1}^3$	$\text{Im } B_{5,1}^3$	$\text{Re } B_{7,1}^3$	$\text{Im } B_{7,1}^3$
0.0	1.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.5	0.94033	-0.12797	-0.03360	-0.23128	-0.02860	0.00429
1.0	0.77238	-0.22544	-0.12356	-0.41192	-0.10454	0.03208
1.5	0.52654	-0.27028	-0.24081	-0.50381	-0.20110	0.09777
2.0	0.24554	-0.25481	-0.34718	-0.49108	-0.28325	0.19830
2.5	-0.02517	-0.18742	-0.40656	-0.39107	-0.32010	0.31468
3.0	-0.25071	-0.09005	-0.39599	-0.20959	-0.29359	0.41627
3.5	-0.39597	0.00823	-0.31256	-0.01307	-0.20468	0.47095
4.0	-0.46452	0.07937	-0.17385	0.16445	-0.07290	0.45556
4.5	-0.46144	0.10383	-0.01282	0.29156	0.07030	0.36442
5.0	-0.40596	0.07534	0.13195	0.35406	0.19162	0.21190
5.5	-0.32146	-0.00309	0.22708	0.34778	0.26608	0.02920
6.0	-0.22876	-0.09788	0.25318	0.31564	0.28406	-0.14438
6.5	-0.14185	-0.19901	0.20902	0.25683	0.25325	-0.27225
7.0	-0.06649	-0.27830	0.11062	0.20120	0.19460	-0.33033
7.5	-0.00194	-0.31910	-0.01477	0.16157	0.13446	-0.31286
8.0	0.05590	-0.31479	-0.13686	0.13929	0.09548	-0.23241
8.5	0.11046	-0.26889	-0.23042	0.12599	0.08934	-0.11486
9.0	0.16190	-0.19266	-0.28093	0.10871	0.11365	0.00926
9.5	0.20612	-0.10106	-0.28625	0.07578	0.15373	0.11336
10.0	0.23599	-0.00884	-0.25453	0.02192	0.18874	0.18145

(continued)

TABLE 3 (continued).

q	$\text{Re } B_{3.2}^3$	$\text{Im } B_{3.2}^3$	$\text{Re } B_{5.2}^3$	$\text{Im } B_{5.2}^3$	$\text{Re } B_{7.2}^3$	$\text{Im } B_{7.2}^3$
0.0	1.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.5	0.97996	0.00069	-0.01009	-0.19764	-0.02250	0.00181
1.0	0.92190	0.00525	-0.03772	-0.37542	-0.08577	0.01340
1.5	0.83151	0.01657	-0.07585	-0.51672	-0.17810	0.04193
2.0	0.71735	0.03577	-0.11444	-0.61057	-0.28337	0.08894
2.5	0.58954	0.06189	-0.14299	-0.65269	-0.38351	0.14992
3.0	0.45825	0.09201	-0.15300	-0.64589	-0.46302	0.21533
3.5	0.33260	0.12185	-0.13987	-0.59860	-0.51158	0.27275
4.0	0.21952	0.14669	-0.10375	-0.52270	-0.52537	0.30990
4.5	0.12352	0.16235	-0.04960	-0.43103	-0.50708	0.31758
5.0	0.04654	0.16597	0.01422	-0.33520	-0.46416	0.29206
5.5	-0.01157	0.15664	0.07774	-0.25136	-0.40647	0.23574
6.0	-0.05227	0.13538	0.13140	-0.16297	-0.34359	0.15659
6.5	-0.07790	0.10498	0.16800	-0.09409	-0.28281	0.06636
7.0	-0.09092	0.06926	0.18381	-0.03726	-0.22792	-0.02219
7.5	-0.09361	0.03248	0.17868	0.00885	-0.17942	-0.09760
8.0	-0.08797	-0.00140	0.15580	0.04565	-0.13546	-0.15198
8.5	-0.07581	-0.02932	0.12040	0.07396	-0.09336	-0.18180
9.0	-0.05882	-0.04937	0.07861	0.09375	-0.05114	-0.18780
9.5	-0.03873	-0.06087	0.03621	0.10449	-0.00850	-0.17394
10.0	-0.01746	-0.06419	-0.00221	0.10562	0.03290	-0.14610

q	$\text{Re } B_{3.3}^3$	$\text{Im } B_{3.3}^3$	$\text{Re } B_{5.3}^3$	$\text{Im } B_{5.3}^3$	$\text{Re } B_{7.3}^3$	$\text{Im } B_{7.3}^3$
0.0	1.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.5	0.96658	0.21912	0.01361	-0.13171	-0.01301	-0.00057
1.0	0.86926	0.42004	0.05307	-0.25006	-0.05028	-0.00411
1.5	0.71659	0.58615	0.11424	-0.34310	-0.10671	-0.01343
2.0	0.52192	0.70411	0.19065	-0.40152	-0.17430	-0.03098
2.5	0.30212	0.76484	0.27407	-0.41969	-0.24369	-0.05808
3.0	0.07591	0.76454	0.35527	-0.39616	-0.30505	-0.09519
3.5	-0.13769	0.70482	0.42492	-0.33372	-0.34945	-0.14143
4.0	-0.32142	0.59259	0.47446	-0.23905	-0.36990	-0.19443
4.5	-0.46108	0.43928	0.49713	-0.12186	-0.36229	-0.25045
5.0	-0.54690	0.25982	0.48850	0.00598	-0.32558	-0.30444
5.5	-0.57435	0.07916	0.44720	0.12708	-0.26198	-0.35075
6.0	-0.54386	-0.10965	0.37517	0.24392	-0.17671	-0.38345
6.5	-0.46312	-0.26643	0.27740	0.33148	-0.07704	-0.39736
7.0	-0.34131	-0.38601	0.16166	0.38681	0.02826	-0.38854
7.5	-0.19299	-0.45912	0.03773	0.40539	0.12988	-0.35510
8.0	-0.03408	-0.48122	-0.08364	0.38644	0.21900	-0.29768
8.5	0.11914	-0.45286	-0.19174	0.33269	0.28792	-0.21940
9.0	0.25174	-0.37945	-0.27715	0.25025	0.33095	-0.12592
9.5	0.35133	-0.27053	-0.33247	0.14771	0.34459	-0.02467
10.0	0.40942	-0.13882	-0.35340	0.03547	0.32814	0.07570

These functions have been computed for $I = \lambda, \lambda+2, \lambda+4$, and $q \leq 10$ and the result is given in Table 3. For odd values of $\lambda+I$ the functions B_{IK}^λ vanish, while for $I < \lambda$ and $K < 0$ they can be found from the symmetry relations

$$\text{and } \left. \begin{aligned} B_{IK}^\lambda &= B_{\lambda K}^I \\ B_{IK}^\lambda &= B_{I-K}^\lambda \end{aligned} \right\} \quad (65)$$

For $I = 0$ the functions B_{IK}^λ are proportional to the functions A_λ which are tabulated in refs. 4 and 8.

$$B_{00}^\lambda(q) = \sqrt{2\lambda+1} A_\lambda(q) \quad (66)$$

and we have therefore omitted the table of B_{00}^2 .

For convenience we have computed also the functions $|B_{IK}^\lambda|^2$ which determine the relative excitation probabilities. They are given for $\lambda = 1, 2$ and 3 and $I \leq \lambda+4$ in Table 4.

TABLE 4.

The quantities $|B_{IK}^\lambda(q)|^2$ for $\lambda = 1, 2$ and 3 and $I \leq \lambda+4$ as functions of q . These quantities determine in the $\chi(\vartheta, \xi)$ approximation, Eq. (37), the relative excitation probability of the rotational states in the coupled band.

q	$ B_{I,0}^1 ^2$			$ B_{I,1}^1 ^2$		
	$I = 1$	$I = 3$	$I = 5$	$I = 1$	$I = 3$	$I = 5$
0.0	1.0000	0.0000	0.0000	1.0000	0.0000	0.0000
0.5	0.9334	0.0656	0.0010	0.9555	0.0439	0.0006
1.0	0.7555	0.2287	0.0154	0.8350	0.1554	0.0093
1.5	0.5221	0.4075	0.0658	0.6727	0.2846	0.0401
2.0	0.2990	0.5158	0.1631	0.5095	0.3773	0.1008
2.5	0.1349	0.5078	0.2887	0.3762	0.4016	0.1822
3.0	0.0468	0.3964	0.3970	0.2878	0.3596	0.2590
3.5	0.0208	0.2400	0.4402	0.2391	0.2810	0.3023
4.0	0.0269	0.1050	0.3964	0.2150	0.2040	0.2967
4.5	0.0371	0.0319	0.2844	0.1995	0.1541	0.2490
5.0	0.0299	0.0208	0.1536	0.1832	0.1358	0.1838
5.5	0.0249	0.0429	0.0556	0.1655	0.1361	0.1296
6.0	0.0147	0.0614	0.0168	0.1468	0.1387	0.1030
6.5	0.0079	0.0599	0.0285	0.1325	0.1320	0.1023
7.0	0.0091	0.0408	0.0584	0.1242	0.1170	0.1126
7.5	0.0111	0.0196	0.0750	0.1181	0.1010	0.1176
8.0	0.0114	0.0094	0.0657	0.1122	0.0906	0.1102
8.5	0.0110	0.0129	0.0397	0.1055	0.0873	0.0942
9.0	0.0070	0.0205	0.0164	0.0984	0.0874	0.0792
9.5	0.0044	0.0246	0.0091	0.0921	0.0859	0.0721
10.0	0.0062	0.0208	0.0169	0.0874	0.0809	0.0729

(continued)

TABLE 4 (continued).

q	$ B_{I,0}^2 ^2$				$ B_{I,1}^2 ^2$		
	$I = 0$	$I = 2$	$I = 4$	$I = 6$	$I = 2$	$I = 4$	$I = 6$
0.0	0.0000	1.0000	0.0000	0.0000	1.0000	0.0000	0.0000
0.5	0.0841	0.8542	0.0607	0.0010	0.9468	0.0524	0.0008
1.0	0.2850	0.5072	0.1937	0.0136	0.8016	0.1866	0.0115
1.5	0.4812	0.1684	0.2919	0.0541	0.6016	0.3450	0.0501
2.0	0.5597	0.0245	0.2776	0.1192	0.3936	0.4622	0.1280
2.5	0.4842	0.1188	0.1667	0.1773	0.2179	0.4944	0.2362
3.0	0.3098	0.3363	0.0562	0.1884	0.0968	0.4358	0.3438
3.5	0.1389	0.4973	0.0462	0.1408	0.0319	0.3168	0.4118
4.0	0.0514	0.4857	0.1584	0.0699	0.0090	0.1847	0.4127
4.5	0.0600	0.3200	0.3173	0.0382	0.0083	0.0806	0.3456
5.0	0.1158	0.1216	0.4073	0.0879	0.0130	0.0236	0.2367
5.5	0.1540	0.0146	0.3605	0.2010	0.0135	0.0104	0.1258
6.0	0.1412	0.0409	0.2129	0.3068	0.0110	0.0165	0.0464
6.5	0.0917	0.1385	0.0614	0.3312	0.0061	0.0264	0.0116
7.0	0.0459	0.2089	0.0018	0.2521	0.0026	0.0276	0.0126
7.5	0.0343	0.1937	0.0522	0.1194	0.0017	0.0200	0.0279
8.0	0.0552	0.1125	0.1494	0.0184	0.0024	0.0100	0.0382
8.5	0.0819	0.0347	0.2050	0.0077	0.0032	0.0038	0.0352
9.0	0.0879	0.0166	0.1756	0.0777	0.0030	0.0032	0.0226
9.5	0.0685	0.0588	0.0901	0.1624	0.0021	0.0057	0.0097
10.0	0.0408	0.1133	0.0186	0.1898	0.0012	0.0077	0.0037

q	$ B_{I,2}^2 ^2$			$ B_{I,0}^3 ^2$			
	$I = 2$	$I = 4$	$I = 6$	$I = 1$	$I = 3$	$I = 5$	$I = 7$
0.0	1.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
0.5	0.9733	0.0264	0.0003	0.0656	0.8734	0.0601	0.0009
1.0	0.8989	0.0963	0.0046	0.2287	0.5611	0.1961	0.0136
1.5	0.7921	0.1860	0.0206	0.4075	0.2250	0.3084	0.0550
2.0	0.6728	0.2672	0.0542	0.5158	0.0258	0.3149	0.1251
2.5	0.5592	0.3178	0.1038	0.5078	0.0334	0.2111	0.1942
3.0	0.4639	0.3298	0.1598	0.3964	0.1958	0.0745	0.2188
3.5	0.3913	0.3092	0.2072	0.2400	0.3835	0.0081	0.1760
4.0	0.3393	0.2708	0.2336	0.1050	0.4741	0.0673	0.0890
4.5	0.3023	0.2306	0.2344	0.0319	0.4215	0.2205	0.0192
5.0	0.2743	0.1989	0.2143	0.0208	0.2702	0.3719	0.0265
5.5	0.2509	0.1778	0.1847	0.0429	0.1099	0.4354	0.1247
6.0	0.2303	0.1665	0.1573	0.0614	0.0239	0.3577	0.2659
6.5	0.2121	0.1578	0.1388	0.0599	0.0221	0.2093	0.3686
7.0	0.1964	0.1488	0.1299	0.0408	0.0688	0.0707	0.3711
7.5	0.1832	0.1386	0.1263	0.0196	0.1080	0.0095	0.2726
8.0	0.1720	0.1285	0.1230	0.0094	0.1063	0.0333	0.1323
8.5	0.1623	0.1198	0.1172	0.0129	0.0694	0.0952	0.0287
9.0	0.1534	0.1132	0.1093	0.0205	0.0280	0.1351	0.0078
9.5	0.1452	0.1080	0.1012	0.0246	0.0098	0.1224	0.0572
10.0	0.1378	0.1035	0.0950	0.0208	0.0194	0.0719	0.1220

(continued)

TABLE 4 (continued).

q	$ B_{I,1}^3 ^2$			
	$I = 1$	$I = 3$	$I = 5$	$I = 7$
0.0	0.0000	1.0000	0.0000	0.0000
0.5	0.0439	0.9006	0.0546	0.0009
1.0	0.1554	0.6474	0.1849	0.0120
1.5	0.2846	0.3503	0.3118	0.0500
2.0	0.3773	0.1252	0.3617	0.1196
2.5	0.4016	0.0358	0.3182	0.2015
3.0	0.3596	0.0710	0.2007	0.2595
3.5	0.2810	0.1569	0.0979	0.2637
4.0	0.2040	0.2221	0.0573	0.2128
4.5	0.1541	0.2237	0.0852	0.1377
5.0	0.1358	0.1705	0.1428	0.0816
5.5	0.1361	0.1034	0.1725	0.0716
6.0	0.1387	0.0619	0.1637	0.0807
6.5	0.1320	0.0597	0.1096	0.1383
7.0	0.1170	0.0819	0.0527	0.1470
7.5	0.1010	0.1018	0.0263	0.1160
8.0	0.0906	0.1022	0.0381	0.0631
8.5	0.0873	0.0845	0.0690	0.0212
9.0	0.0874	0.0633	0.0907	0.0130
9.5	0.0859	0.0527	0.0877	0.0365
10.0	0.0809	0.0558	0.0652	0.0686

q	$ B_{I,2}^3 ^2$			$ B_{I,3}^3 ^2$		
	$I = 3$	$I = 5$	$I = 7$	$I = 3$	$I = 5$	$I = 7$
0.0	1.0000	0.0000	0.0000	1.0000	0.0000	0.0000
0.5	0.9603	0.0392	0.0005	0.9823	0.0175	0.0002
1.0	0.8499	0.1424	0.0075	0.9320	0.0654	0.0025
1.5	0.6917	0.2728	0.0335	0.8571	0.1308	0.0116
2.0	0.5159	0.3859	0.0882	0.7682	0.1976	0.0313
2.5	0.3514	0.4464	0.1696	0.6763	0.2512	0.0628
3.0	0.2185	0.4406	0.2608	0.5903	0.2832	0.1021
3.5	0.1255	0.3779	0.3361	0.5157	0.2919	0.1421
4.0	0.0697	0.2840	0.3720	0.4545	0.2823	0.1746
4.5	0.0416	0.1882	0.3580	0.4056	0.2620	0.1940
5.0	0.0297	0.1126	0.3007	0.3666	0.2387	0.1987
5.5	0.0247	0.0692	0.2208	0.3361	0.2161	0.1917
6.0	0.0211	0.0438	0.1426	0.3078	0.2002	0.1783
6.5	0.0171	0.0371	0.0844	0.2855	0.1868	0.1638
7.0	0.0131	0.0352	0.0524	0.2655	0.1758	0.1518
7.5	0.0098	0.0320	0.0417	0.2480	0.1658	0.1430
8.0	0.0077	0.0264	0.0414	0.2327	0.1563	0.1366
8.5	0.0066	0.0200	0.0418	0.2193	0.1474	0.1310
9.0	0.0059	0.0150	0.0379	0.2074	0.1394	0.1254
9.5	0.0052	0.0122	0.0303	0.1966	0.1324	0.1194
10.0	0.0044	0.0112	0.0224	0.1869	0.1262	0.1134

For a more accurate description of the dependence of the excitation amplitudes on the deflection angle one needs the functions $B_{IK}^{\lambda\mu}$ with $\mu \neq 0$. These functions satisfy a number of symmetry relations which are consequences of symmetry properties of the 3- j symbols and of the fact that A_J vanishes for odd values of J . These relations are

$$B_{IK}^{\lambda\mu} = B_{I\mu}^{\lambda K} = B_{\lambda K}^{I\mu} \quad (67)$$

and

$$B_{IK}^{\lambda\mu} = (-1)^{I-\lambda} B_{IK}^{\lambda-\mu} = B_{I-K}^{\lambda-\mu}. \quad (68)$$

For $\lambda = 1$ and 2 Eq. (26) reads explicitly

$$B_{I,1}^{1,1} = \left\{ \begin{array}{l} \sqrt{\frac{3}{4(2I+1)}} [(I+1)A_{I-1} + IA_{I+1}] \quad I \text{ odd} \\ \sqrt{\frac{3(2I+1)}{4}} A_I \quad I \text{ even} \end{array} \right\} \quad (69)$$

$$B_{I,1}^{2,1} = \left\{ \begin{array}{l} \sqrt{\frac{5(2I+1)}{4}} \left[\frac{2(I-1)(I+1)}{(2I-1)(2I+1)} A_{I-2} \right. \\ \quad \left. + \frac{3}{(2I-1)(2I+3)} A_I + \frac{2I(I+2)}{(2I+1)(2I+3)} A_{I+2} \right] \quad I \text{ even} \\ \sqrt{\frac{5}{4(2I+1)}} [(I-1)A_{I-1} + (I+2)A_{I+1}] \quad I \text{ odd} \end{array} \right.$$

$$B_{I,2}^{2,1} = \left\{ \begin{array}{l} \sqrt{\frac{5(I-1)(I+2)(2I+1)}{4}} \left[\frac{I+1}{(2I-1)(2I+1)} A_{I-2} \right. \\ \quad \left. - \frac{3}{(2I-1)(2I+3)} A_I - \frac{I}{(2I+1)(2I+3)} A_{I+2} \right] \quad I \text{ even} \\ \sqrt{\frac{5(I-1)(I+2)}{4(2I+1)}} [A_{I-1} - A_{I+1}] \quad I \text{ odd} \end{array} \right\} \quad (70)$$

$$B_{I,2}^{2,2} = \left\{ \begin{array}{l} \sqrt{\frac{5(2I+1)}{16}} \left[\frac{(I+1)(I+2)}{(2I-1)(2I+1)} A_{I-2} \right. \\ \quad \left. + \frac{6(I-1)(I+2)}{(2I-1)(2I+3)} A_I + \frac{(I-1)I}{(2I+1)(2I+3)} A_{I+2} \right] \quad I \text{ even} \\ \sqrt{\frac{5}{4(2I+1)}} [(I+2)A_{I-1} + (I-1)A_{I+1}]. \quad I \text{ odd} \end{array} \right.$$

The absolute squares of the functions $B_{IK}^{2\mu}$ have been computed numerically and the result is given in Table 5.

We note finally that the functions $B_{IK}^{\lambda\mu}$ satisfy the relations

$$B_{IK}^{\lambda\mu}(0) = \delta_{I,\lambda} \quad (71)$$

and (cf. Eq. (31))

$$\sum_I B_{IK}^{\lambda'\mu*}(q) B_{IK}^{\lambda\mu}(q) = \delta_{\lambda',\lambda}. \quad (72)$$

TABLE 5.

The quantities $|B_{IK}^{\lambda\mu}(q)|^2$ for $\lambda = 2$, $\mu \neq 0$, $K \neq 0$, and $I \leq 6$ as functions of q . From these quantities the relative quadrupole excitation probabilities can be determined by means of Eq. (34). In the application of the table one should employ the symmetry relations (67)–(68). For $K = 0$ or $\mu = 0$ the functions reduce to the quantities $|B_{IK}^{\lambda}|^2$ given in Table 4.

$$|B_{I,1}^{2,1}|^2$$

q	$I = 1$	$I = 2$	$I = 3$	$I = 4$	$I = 5$	$I = 6$
0.0	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
0.5	0.0631	0.8811	0.0126	0.0423	0.0003	0.0006
1.0	0.2138	0.5879	0.0484	0.1370	0.0042	0.0083
1.5	0.3609	0.2708	0.1005	0.2122	0.0187	0.0332
2.0	0.4198	0.0737	0.1548	0.2120	0.0487	0.0738
2.5	0.3632	0.0513	0.1921	0.1393	0.0918	0.1108
3.0	0.2324	0.1496	0.1958	0.0521	0.1368	0.1191

$$|B_{I,2}^{2,1}|^2$$

q	$I = 2$	$I = 3$	$I = 4$	$I = 5$	$I = 6$
0.0	1.0000	0.0000	0.0000	0.0000	0.0000
0.5	0.9470	0.0305	0.0217	0.0005	0.0002
1.0	0.8048	0.1079	0.0762	0.0072	0.0035
1.5	0.6157	0.1976	0.1373	0.0312	0.0152
2.0	0.4300	0.2620	0.1771	0.0784	0.0380
2.5	0.2861	0.2789	0.1798	0.1417	0.0682
3.0	0.1988	0.2497	0.1482	0.2014	0.0955

$$|B_{I,2}^{2,2}|^2$$

q	$I = 2$	$I = 3$	$I = 4$	$I = 5$	$I = 6$
0.0	1.0000	0.0000	0.0000	0.0000	0.0000
0.5	0.9125	0.0751	0.0114	0.0009	0.0001
1.0	0.6860	0.2539	0.0458	0.0124	0.0017
1.5	0.4087	0.4269	0.1003	0.0515	0.0090
2.0	0.1753	0.4924	0.1634	0.1234	0.0284
2.5	0.0419	0.4179	0.2135	0.2075	0.0658
3.0	0.0075	0.2538	0.2278	0.2658	0.1175

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